Goals of the Summer:

- To understand the classical theory of optical coherence for the sake of illuminating the quantum behavior of light which we plan to investigate throughout the school year.

- In particular, to recreate the measurements of Arecchi *et al.* for the degree of second-order coherence

$$g^{(2)}(\tau) = \frac{\langle \bar{I}(t)\bar{I}(t+\tau)\rangle}{\bar{I}^2} = \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle}$$

for thermal and laser light.

Prior to considering the progress of this project, we require some background relating to the theory of optical coherence including:

- a model for a chaotic source of light,
- a classical formulation of the function of beam splitters,
- the definition and properties of the degree of first-order coherence,
- and the definition and properties of the degree of second-order coherence

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Model of a Chaotic Source:

We note that we require a model of a chaotic source for the sake of predicting the degree of second-order coherence for thermal light.

•Consider a source that consists of excited atoms that emit wave trains of constant amplitude and frequency.

•Suppose that each atom steadily emits radiation until a collision occurs that temporarily interrupts the radiation via shifting the energy levels of the atom.

•Assume that the collision only changes the phase of the radiation and not its frequency

- •Then, we find that the phase of the radiation that each atom emits becomes a function of time as it changes abruptly while its frequency remains constant.
- •Thus, collisions partition the wave trains of the atom into finite sections that yield a spectrum of frequencies upon considering their Fourier decompositions.

•Assuming the wave trains from the atoms exhibit linear polarization along some fixed axis, we write the electric field that the source generates as

$$
\begin{aligned}
E(t) &= E_1(t) + E_2(t) + ... + E_n \\
&= E_0 \exp(-i\omega_0 t)[\exp(i\phi_1(t)) + \exp(i\phi_2(t)) + ... + \exp(i\phi_n(t)) \\
&= E_0 \exp(-i\omega_0 t) a(t) \exp(i\phi(t))
\end{aligned}
$$

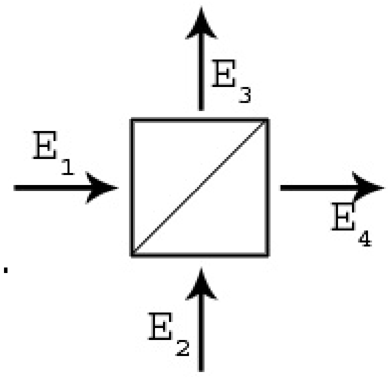
Classical Formulation of the Beam Splitter:

As we consider the Mach-Zender interferometer as a means of introducing the degree of first-order coherence, we must formulate the function of the beam splitter classically.

•Heeding the adjacent figure, we relate the fields at the outputs to those at the inputs via

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = \begin{pmatrix} \mathcal{R}_{31} & \mathcal{T}_{32} \\ \mathcal{T}_{41} & \mathcal{R}_{42} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$

where the coefficients denote reflection and transmission coefficients.



•From the conservation of energy between the light at the inputs and outputs, we readily derive the following conditions on the coefficients

$$\phi_{31} + \phi_{42} - \phi_{32} - \phi_{41} = \pm\pi \text{ and } |\mathcal{R}_{31}| = |\mathcal{R}_{42}| = |\mathcal{R}| \text{ and } |\mathcal{T}_{32}| = |\mathcal{T}_{41}| = |\mathcal{T}|$$

where note that we separate the coefficients into amplitude and phase factors.

$$\mathcal{R}_{31} = |\mathcal{R}_{31}|\exp(i\phi_{31})$$

•In our work, we assume that

$$\mathcal{R}_{31} = \mathcal{R}_{42} \equiv \mathcal{R} = |\mathcal{R}|\exp(i\phi_{\mathcal{R}}) \text{ and } \mathcal{T}_{32} = \mathcal{T}_{41} \equiv \mathcal{T} = |\mathcal{T}|\exp(i\phi_{\mathcal{T}})$$

thus allowing the relations above to reduce to

$$|\mathcal{R}|^2 + |\mathcal{T}|^2 = 1 \text{ and } \mathcal{R}\mathcal{T}^* + \mathcal{T}\mathcal{R}^* = 0 \text{ and } \phi_{\mathcal{R}} - \phi_{\mathcal{T}} = \pm\pi/2$$

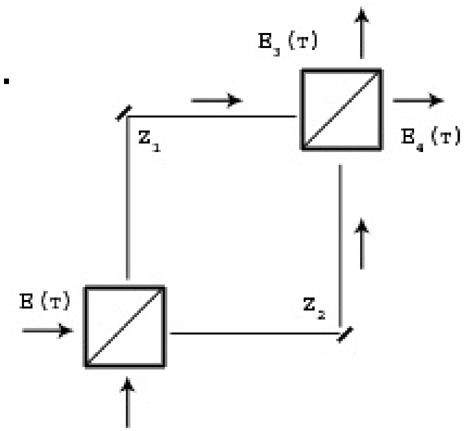
•Moreover, we often consider a 50-50 beam splitter where in particular

$$|\mathcal{R}| = |\mathcal{T}| = 1/\sqrt{2} \text{ with } \phi_{\mathcal{R}} - \phi_{\mathcal{T}} = \pi/2$$

Analysis of the Mach-Zender Interferometer:

To motivate the origin of the degree of first-order coherence, we consider the source of interference in the output of the Mach-Zender Interferometer.

•Suppose that chaotic light of linear polarization impinges on arm 1 of the interferometer to the right while no light is incident on arm 2.



•Taking the reflection and transmission coefficients as constant – despite their dependence on frequency and the spread of frequencies in chaotic light – we write the field in arm 4 as

$$E_4(t) = \mathcal{RT}E(t_1) + \mathcal{TR}E(t_2)$$

•The intensity of the light at the output – which we write as the average over a cycle of oscillation since we cannot practically resolve the oscillations of the field at the frequency of the light – is

$$\begin{aligned} \bar{I}_4(t) &= \frac{1}{2}\epsilon_0 c|E_4(t)|^2 \\ &= \frac{1}{2}\epsilon_0 c|\mathcal{R}|^2|\mathcal{T}|^2\left[|E(t_1)|^2 + |E(t_2)|^2 + 2\mathrm{Re}\left(E^*(t_1)E(t_2)\right)\right] \end{aligned}$$

•In fact, we cannot practically resolve the fringes that result from interference on a scale that compares to the coherence time of the chaotic light, so we average this intensity over a period T that far exceeds the coherence time to yield:

$$\langle\bar{I}_4(t)\rangle = \frac{1}{2}\epsilon_0 c|\mathcal{R}|^2|\mathcal{T}|^2\left[\langle|E(t_1)|^2\rangle + \langle|E(t_2)|^2\rangle + 2\mathrm{Re}\langle E^*(t_1)E(t_2)\rangle\right]$$

The Degree of First-Order Coherence:

In fact, the term dictating the fringes in the interference pattern contains the first-order correlation function from which we define the degree of first-order coherence.

•We define the first-order correlation function explicitly as a time average

$$\langle E^*(t_1)E(t_2)\rangle = \frac{1}{T}\int_T E^*(t_1)E(t_2)\mathrm{d}t_1$$

where note that the integrand contains a single variable as the times differ by a constant.

•For a stationary source – where the influences that control the fluctuations in the light remain constant – the time average simplifies to a statistical average as the correlation function does not depend on the starting time of T and T samples all values of the electric field at times t_1 and t_2.

•As the correlation function depends only on the delay between the times of measurement for a stationary source, we rewrite the correlation function as

$$\langle E^*(t)E(t+\tau)\rangle = \frac{1}{T}\int_T E^*(t)E(t+\tau)$$

•We then define the degree of first-order coherence via normalizing the correlation function:

$$\boxed{g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle E^*(t)E(t)\rangle}}$$

Properties of The Degree of First-Order Coherence:

Some properties of the degree of first-order coherence arise immediately when we consider stationary sources where the correlation is only a function of the delay.

•For a stationary source where the correlation function depends only on the delay:

$$\langle E^*(t)E(t-\tau)\rangle = \langle E^*(t+\tau)E(t)\rangle = \langle E^*(t)E(t+\tau)\rangle^*$$

and so

$$g^{(1)}(-\tau) = g^{(1)}(\tau)^*$$

•For a chaotic source, we rewrite the correlation function as

$$\langle E^*(t)E(t+\tau)\rangle = E_0^2 \exp(-i\omega_0\tau)\langle(\exp(-i\phi_1(t)) + ... + \exp(-i\phi_n(t)))\,(\exp(i\phi_1(t+\tau)) + ... + \exp(i\phi_n(t+\tau)))\rangle$$

where the cross terms contribute nothing as they correlate to products of the phase angles of the wave trains of pairs of atoms which are unrelated.

•The remaining terms yield

$$\begin{aligned}\langle E^*(t)E(t+\tau)\rangle &= E_0^2 \exp(-i\omega_0\tau)\sum_{i=1}^{n}\langle \exp\left(i(\phi_i(t+\tau)-\phi_i(t))\right)\rangle \\ &= n\langle E_i^*(t)E_i(t+\tau)\rangle\end{aligned}$$

since the atoms are equivalent.

Generalization of The Degree of First-Order Coherence:

Upon generalizing the definition of the degree of first-order coherence, we consider light at a pair of points in space-time according to the value of the degree of first-order coherence

•To generalize the definition of the degree of first-order coherence for the correlation between fields at arbitrary points (z_1,t_1) and (z_2,t_2) in space-time, we write

$$g^{(1)}\left((z_1,t_1),(z_2,t_2)\right) = \frac{\langle E^*(z_1,t_1)E(z_2,t_2)\rangle}{\left(\langle|E(z_1,t_1)|^2\rangle\langle|E(z_2,t_2)|^2\rangle\right)^{1/2}}$$

•We then characterize light at two points in space-time as follows:

$$\text{For } |g^{(1)}\left((z_1,t_1),(z_2,t_2)\right)| \begin{bmatrix} = & 1 \\ = & 0 \\ \neq & 0 \text{ or } 1 \end{bmatrix} \text{ the light is } \begin{bmatrix} \text{first-order coherent} \\ \text{incoherent} \\ \text{partially coherent} \end{bmatrix}$$

•We note that since the fluctuations propagate at speed c without changing their form, we easily revert this generalized definition to our earlier definition via letting

$$\tau = t_2 - t_1 - \frac{z_2 - z_1}{c}$$

• Then, from its definition we clearly find that

$$g^1(0) = 1 \text{ and } g^1(0) \approx 1 \text{ for } \tau << \tau_c$$

and we also note that

$$g^{(1)}(\tau) \longrightarrow 0 \text{ for } \tau >> \tau_c$$

The Degree of Second-Order Coherence:

Having discussed the degree of first-order coherence, we readily define the degree of second-order coherence and introduce some of its properties as analogues of the former.

- We initially consider pairs of cycle-averaged intensities from a beam of a fixed polarization that we measure at a pair of times at a fixed point in space.
  - The time average – which we again assume to equal a statistical average – of the pairs of readings yields a correlation function for the intensities.

- We then define the degree of second-order coherence via normalizing this correlation function:

$$g^{(2)}(\tau) = \frac{\langle \bar{I}(t)\bar{I}(t+\tau)\rangle}{\bar{I}^2} = \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle}$$

- Analogous to the degree of first-order coherence, we find that

$$g^{(2)}(-\tau) = g^{(2)}(\tau)$$

- Then, from Cauchy's inequality we easily show that

$$\left(\frac{\bar{I}(t_1)+\bar{I}(t_2)+...+\bar{I}(t_N)}{N}\right)^2 \leq \frac{\bar{I}(t_1)^2+\bar{I}(t_2)^2+...+\bar{I}(t_N)^2}{N}$$

which implies that

$$\bar{I}^2 \equiv \langle \bar{I}(t)\rangle^2 \leq \langle \bar{I}(t)^2\rangle$$

and so – since we cannot establish an upper limit – the degree of second-order coherence satisfies

$$1 \leq g^{(2)}(0) \leq \infty$$

More on the Degree of Second-Order Coherence:

•Although we cannot restrict the value for non-zero delays, we find another property – again using the Cauchy inequality – from the following

$$\left(\bar{I}(t_1)\bar{I}(t_1+\tau)+...+\bar{I}(t_N)\bar{I}(t_N+\tau)\right)^2 \leq \left(\bar{I}(t_1)^2+...+\bar{I}(t_N)^2\right)\left(\bar{I}(t_1+\tau)^2+...+\bar{I}(t_N+\tau)^2\right)$$

where the summations on the right are approximately equal when N is large, thus yielding

$$\langle \bar{I}(t)\bar{I}(t+\tau)\rangle \leq \langle \bar{I}(t)^2\rangle$$

which gives

$$g^{(2)})(\tau) \leq g^{(2)}(0)$$

•Analogous to the degree of first-order coherence, we readily generalize the degree of second-order coherence to consider intensity correlations of light between pairs of points in space-time.

- •And, as before – in the case of a stationary source – we can revert this definition to our definition via noting the relationship between times of measurement.

•When both the degree of second-order coherence and the absolute value of the degree of first-order coherence are 1, the light is second-order coherent.

The Degree of Second-Order Coherence for a Thermal Source:

Having defined the degree of second-order coherence and described its properties, we readily apply our model of a chaotic source to predict its values for thermal light.

•Recalling our model for a chaotic source, we write the correlation function as

$$\sum_{i=1}^{n}\langle E_i^*(t)E_i^*(t+\tau)E_i(t+\tau)E_i(t)\rangle + \sum_{i\neq j}\left(\langle E_i^*(t)E_j^*(t+\tau)E_j(t+\tau)E_i(t)\rangle + \langle E_i^*(t)E_j^*(t+\tau)E_i(t+\tau)E_j(t)\rangle\right)$$

where we retain only those terms in which we multiply the field of each atom by its complex conjugate. The other terms vanish as the phases are unrelated.

•Because of the equivalence of the fields that the atoms contribute, this simplifies to

$$n\langle E_i^*(t)E_i^*(t+\tau)E_i(t+\tau)E_i(t)\rangle + n(n-1)\left(\langle E_i^*(t)E_i(t)\rangle^2 + |\langle E_i^*(t)E_i(t+\tau)\rangle|^2\right)$$

which we approximate – for a large number of atoms – as

$$\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle = n^2\left(\langle E_i^*(t)E_i(t)\rangle^2 + |\langle E_i^*(t)E_i(t+\tau)\rangle|^2\right)$$

•Heeding these and earlier results, we readily show that

$$g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2$$

which indicates the limits of the degree of second-order coherence for a thermal source:

$$g^{(2)}(0) = 2 \text{ and } g^{(2)}(\tau) \longrightarrow 1 \text{ for } \tau >> \tau_c$$

•In contrast, for a coherent source, we easily show that

$$g^{(2)}(\tau) = 1$$

Measurement of the Degree of Second-Order Coherence:

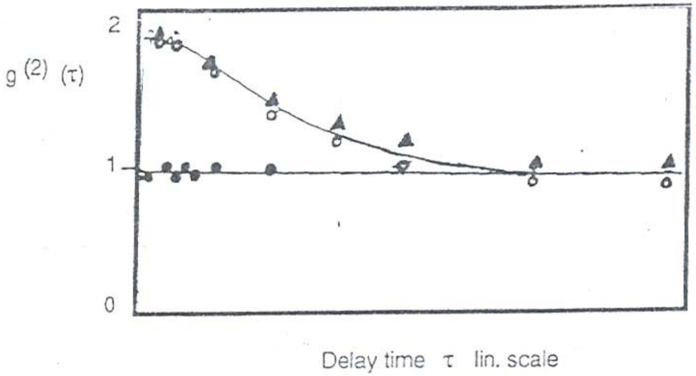
The results that we have established so far indicate the provenience of the degree of second-order coherence. We now consider methods of measuring its value experimentally.

•Experimentally calculating the degree of second-order coherence amounts to counting intensity coincidences between two beams of light over a range of delays.

•We can use a beam splitter to generate beams of light that we can detect individually.
- •Via translating one of the detectors, we introduce a delay.
- •We then normalize the measurements using the results for long delays.

•In such an experiment, the detection time cannot exceed the coherence time of the light.
- •As a result, we encounter problems when measuring coincidences for thermal light where the response of the detectors exceeds the coherence time.
- •Arecchi *et al.* overcame this problem via scattering light from a laser through randomly and rapidly moving objects to simulate a thermal source.

•Results of the Arecchi *et al*. experiment:



Our Apparatus:

Our primary experiment over the summer was to recreate the results that Arecchi *et al.* attained in 1966.

•We direct a He-Ne 632.8 nm laser through an oscillating piece of glass to simulate a thermal source.

•The components between the laser and the beam splitter – including a polarizer and lens – assist in preventing the detectors from saturating and limiting inconsistencies.

•The light then impinges on the beam splitter and the outputs strike detectors that yield analog signals.

•We send the signals from the detectors through buffers to limit issues relating to impedances.

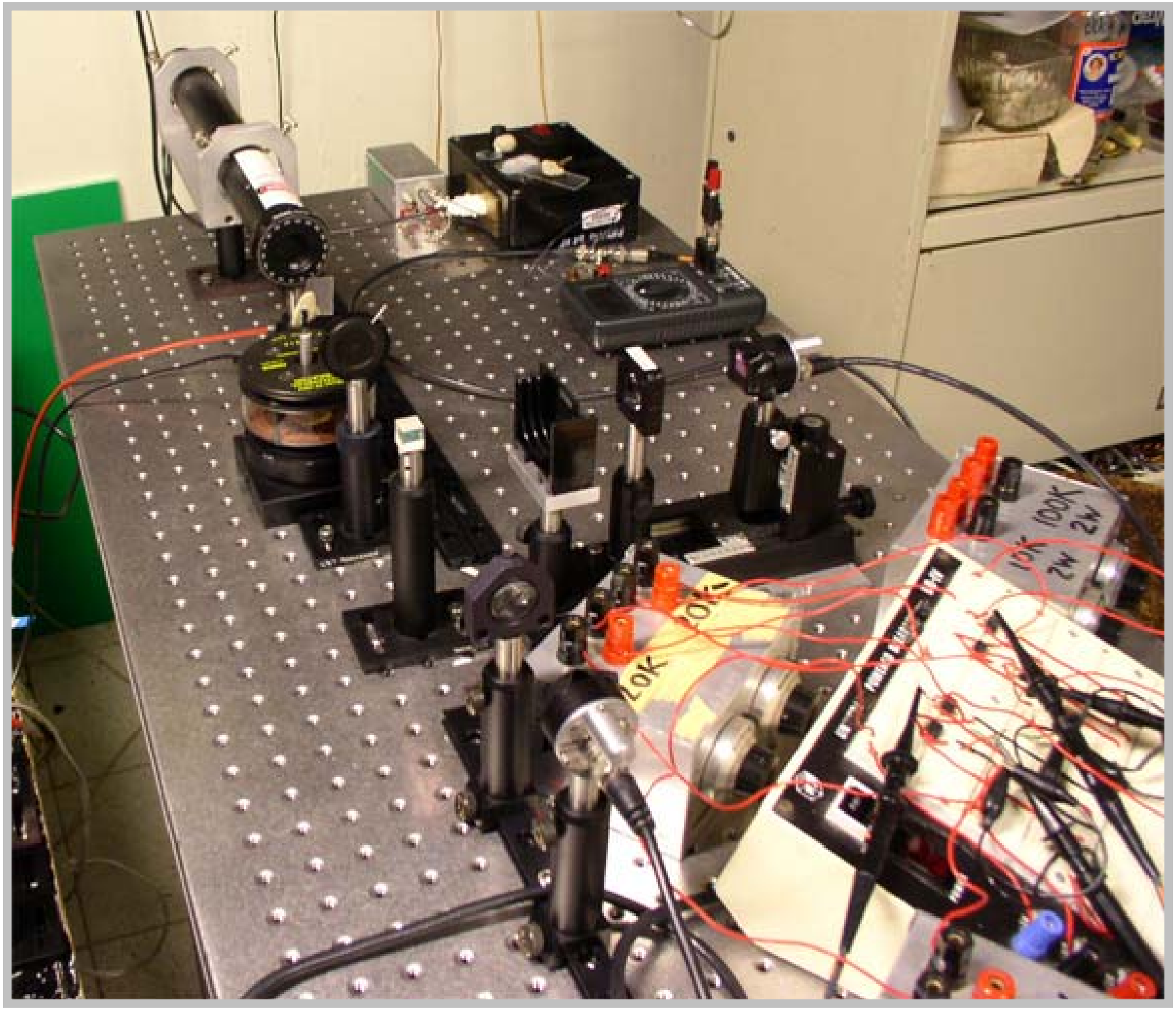
•We then direct the signals through voltage comparators that function as analog-to-digital converters.

- •The comparators enable us to set a window at which to produce digital signals.

•We send one of these signals through a delay generator that we use to electronically control the delay time between the signals.

•At this point we count coincidences between the outputs of the beam splitter using a coincidence counter.

Picture of the Apparatus:



Status of the Experiment:

We have still not attained the results that we seek, although we believe that we are within a week of completing the experiment.

•We have recorded coincidences between the detectors for a range of delay times, but we have not yet attained the pattern that we expect.

•We are unsure of the coherence time of the pseudo-thermal source, and hence we somewhat arbitrarily set the scale of the delay times.

•Given the trends in the data that we collected most recently, we suspect the possibility that some piece of equipment is malfunctioning.

•We plan a more precise method of estimating the coherence time of the pseudo-thermal source and fixing aspects of the experiment that are leading to inconsistencies in the data.

Plans for the Future:

Besides providing invaluable experience with methods in the laboratory, this project will transition into a series of experiments for my thesis that investigate the quantum behavior of light.

- The results that we intend to attain for the degree of second-order coherence of thermal light suggest that the photons arrive in bunches.
  - In particular, the probability that photons will arrive is large for short time delays.
  - In contrast, coherent light is not bunched as the rate that we measure coincidences remains constant.

- These results suggest the possibility of anti-bunching light such that the photons some distance between each other.
  - Such a result, however, would violate the classical conditions that we established earlier in paper as the degree of second-order coherence for a delay of zero would equal one.

- This year, we intend on performing experiments that illustrate the quantum behavior of light. In particular, we hope to:
  - observe entanglement of photons,
  - show the existence of photons (via anti-bunching photons),
  - experimentally verify Bell's theorem,
  - recreate an early experiment showing quantum teleportation.

Acknowledgements

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