Department of Mathematics

November 4, 2019

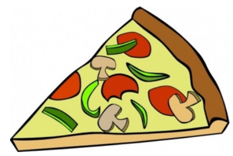
UNDERGRADUATE MATH SEMINAR

The next (and final) math seminar of the fall term will be …

DATE: **Friday, November 8**

Time & **12:30 pm** – Pizza and drinks

Location: **1:00 pm** – Seminar in **Bailey 201**



Professor Casian Pantea

In this week's seminar, **Professor Casian Pantea**, an applied mathematician from the Department of Mathematics at **West Virginia University** will deliver the following talk:

Title: Injectivity of polynomial maps and multistationarity in reaction networks

Abstract: The capacity of biochemical reaction networks to operate at different steady states is crucial in important biological processes like cell division, differentiation, or apoptosis. In this talk we attack the question "when can a certain reaction network admit two or more positive steady states?", i.e., when can the network be multistationary? This boils down to a difficult question about solutions of some high-dimensional polynomial systems, further complicated by the lack of information on coefficient values. However, it turns out that a lot can be said on multistationarity of reaction networks by studying the injectivity of the corresponding polynomial maps. We will survey some classical and some new results on the topic, and illustrate them using relevant biological examples.

Another Interesting *Quanta Magazine* Article

In last week's math newsletter, the opening paragraphs of a recent article in the online math and science *Quanta Magazine* was reproduced. Based on the positive feedback from readers of the newsletter, here is an excerpt from another *Quanta* article, posted October 21, also by Kevin Hartnett.

Mathematicians Begin to Tame Wild 'Sunflower' Problem:

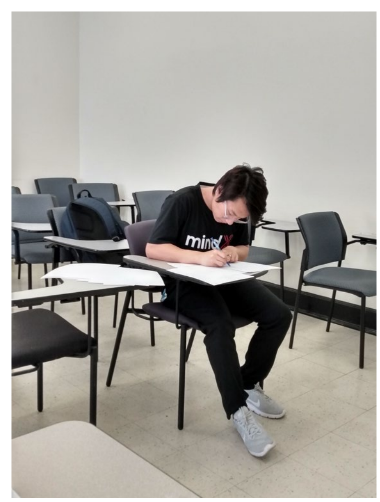
A major advance toward solving the 60-year-old sunflower conjecture is shedding light on how order begins to appear as random systems grow in size.

How do mathematical sunflowers emerge from random data?

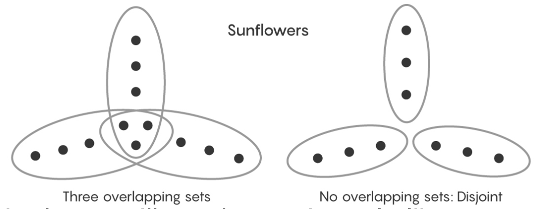
A team of mathematicians and computer scientists has finally made progress on a seemingly simple problem that has bedeviled researchers for nearly six decades.

Posed by the mathematicians Paul Erdős and Richard Rado in 1960, the problem concerns how often you would expect to find patterns resembling sunflowers in large collections of objects, such as a large scattering of points in the plane. While the new result doesn't fully solve Erdős and Rado's sunflower conjecture, it advances the mathematical understanding of how surprisingly intricate structures emerge out of randomness. To do so, it reimagined the problem in terms of a computer function — taking advantage of the increasingly rich interplay between theoretical computer science and pure mathematics. (Continued on next page.)

Seen in Bailey Hall



On Saturday, October 26, 2019, Union College students had the opportunity to participate in the 41st annual Virginia Tech Regional Math Contest. Pictured above is **Son Nguyen,** hard at work, representing Union!



"The paper is a new manifestation of a mathematical idea that's going to be a central idea of our time. The result itself is spectacular," said Gil Kalai of the Hebrew University of Jerusalem.

The sunflower conjecture is about sets, which are collections of objects. The conjecture is easiest to visualize if you think of sets of points in the flat *xy*-plane. First decide on a fixed number of points you want to include in each set. Then draw loops at random so that each loop, or set, encompasses that number of points. It's OK if the loops overlap, so some points may end up inside more than one set, like the intersections in a Venn diagram.

If you draw many loops containing a large number of points, most of the loops will overlap and tangle like a snarl of brambles. But Erdős and Rado predicted that a delicate structure would invariably arise: Three or more sets would all partly overlap each other at exactly the same subset of points, and none … would overlap any other sets.

If you were to delete that common subset of points, the three sets would be arrayed around a void, completely separate from each other — like petals around the dark center of a sunflower. For the purposes of the problem, the simplest kind of sunflower is considered to be one with three sets that don't overlap each other or any other sets; these islands are called "disjoint" sets.

Erdős and Rado conjectured that as you draw more loops, a sunflower inevitably emerges, either as disjoint sets or as sets that overlap in just the right way. Their sunflower conjecture is part of a broader area of mathematics called Ramsey theory, which studies how order begins to appear as random systems grow larger.

Erdős and Rado wanted to know precisely how many sets — of precisely what size — you need in order to be guaranteed a sunflower. They made a modest first step toward solving the problem by establishing a parameter, *w*, that stood for the number of points in each set. The pair then proved you need about w^w sets of size *w* to be sure to find a sunflower made of three sets. So, if you want each set to contain 100 points, they proved you need on the order of 100^{100} sets to be guaranteed a sunflower.

At the same time, Erdős and Rado conjectured that the actual number of sets required to guarantee a sunflower is much smaller than w^w — it's more like a constant number to the *w* power (so 3^w or 80^w or $5{,}000{,}000^w$). Yet they couldn't find a way to prove their intuition was correct.

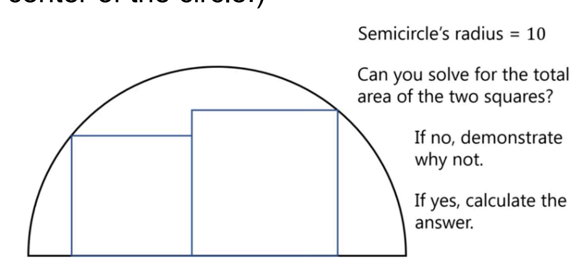
"They said this problem looks extremely simple and were wondering why they couldn't make progress on it," [Ryan] Alweiss of Princeton University said.

They weren't the only ones. Between Erdős and Rado's first result and this new proof 60 years later, only two mathematicians made any progress on the question at all — and they only made incremental advances, one in 1997 and the other earlier this year. [To read the rest of the article, go to Quanta Magazine's website!]

Problem of the Newsletter – November 4, 2019

Last week's problem: Congratulations to **Son Nguyen '23** for solving last week's problem. A solution has been posted at the newsletter sites in Bailey Hall.

This week's problem: The following problem popped up on my Facebook feed this past week. Let's take it as this week's newsletter problem. Two adjacent squares are inscribed in a semicircle of radius 10 as shown. Can you solve for the total area of the two squares? (Note: the point on the diameter that is common to both squares is NOT necessarily the center of the circle.)



Professor Friedman (friedmap@union.edu) will accept solutions until noon on Friday, November 8.