

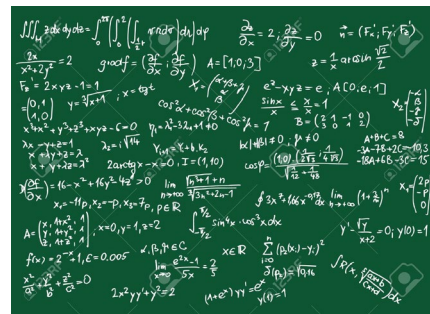
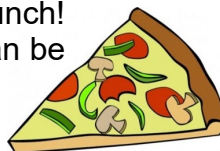
UNDERGRADUATE MATH SEMINAR

This week's seminar is **TUESDAY** during common lunch! Details, including speaker name, title and abstract can be found on the posters around Bailey Hall.

DATE: **Tuesday, March 3**

Time & **12:30 pm** – Pizza, drinks in Bailey 204

Location: **12:45 pm** – Seminar in **Bailey 207**



Some Math in the News

In this feature, we highlight some math that has made it into the popular press.

- Do you sometimes forget the quadratic formula? Are you looking for a geometrically inspired method to solve quadratic equations? Look no further! A recent article in the **New York Times** describes such an approach in “This Professor’s ‘Amazing’ Trick Makes Quadratic Equations Easier: Looking for the answers to $ax^2+bx+c=0$? A mathematician has rediscovered a technique that the ancient Babylonians used.” (<https://www.nytimes.com/2020/02/05/science/quadratic-equations-algebra.html>.) It begins...

“The quadratic equation has frustrated math students for millennia. But a math professor at Carnegie Mellon University in Pittsburgh may have come up with a better way of solving it.

“When I stumbled on this, I was just completely shocked,” said the professor, Po-Shen Loh.

Quadratics, which are introduced in elementary algebra classes, pop up often in physics and engineering in the calculating of trajectories, even in sports. If, while watching the Super Bowl, you had wanted to estimate how far a pass thrown by Patrick Mahomes traveled through the air, you would have been solving a quadratic equation. The equations also show up in calculations for maximizing profit, a key consideration for anyone who wants to succeed in business.

Dr. Loh has not discovered something entirely new. Indeed, his method mixes together ideas dating back thousands of years to the Babylonians. But this is not how modern algebra textbooks present the topic.

“To find out that there’s this trick from thousands of years ago that you can import into here is amazing to me,” Dr. Loh said. “I wanted to share that as widely as possible.”

- Quanta Magazine’s**, “Mathematicians Cut Apart Shapes to Find Pieces of Equations: New work on the problem of ‘scissors congruence’ explains when it is possible to slice up one shape and reassemble it as another” highlights work by Jonathan Campbell of Duke (who spoke in a relatively recent Union College Math Seminar on this topic!) and a colleague at Cornell. <https://www.quantamagazine.org/mathematicians-cut-apart-shapes-to-find-pieces-of-equations-20191031/>

For example, in this parabola:

$$y = x^2 - 4x - 5$$

The **two solutions** when $y = 0$ are the symmetrical points r and s , where the parabola crosses the x-axis.

The midpoint, or average, of r and s is the **axis of symmetry** of the parabola. We want $r + s = -b$, which happens when the average of r and s is $-b/2$. In this example: $4/2 = 2$.

The two solutions to the quadratic equation will be the axis of symmetry plus or minus an **unknown amount**, which we’ll call u . In this example:

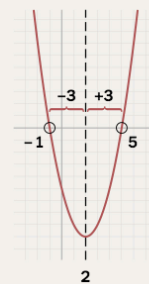
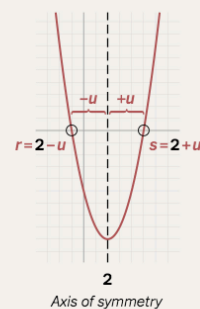
$$r = 2 - u \text{ and } s = 2 + u$$

To find u , we want the product of r and s to be equal to c , which in this example is -5 . Rewriting r and s in terms of u :

$$r \times s = -5 \\ (2 - u) \times (2 + u) = -5$$

Solving that yields $2^2 - u^2 = -5$ or $u^2 = 9$, so $u = 3$ works.

The **two solutions** to this quadratic equation are $2 - u$ and $2 + u$, or -1 and 5 . In other words, this parabola intersects the x-axis when $x = -1$ and $x = 5$.



An example of the rediscovered method of solving a quadratic equation, exploiting the symmetry of parabolas.

If you have two flat paper shapes and a pair of scissors, can you cut up one shape and rearrange it as the other? If you can, the shapes are “scissors congruent.”

But, mathematicians wonder, can you tell if two shapes share this relationship even without using scissors? In other words, are there characteristics of each shape you could measure ahead of time to determine whether they’re scissors congruent?

For two-dimensional shapes, the answer is easy: Just determine their areas. If they’re the same, the shapes are scissors congruent.

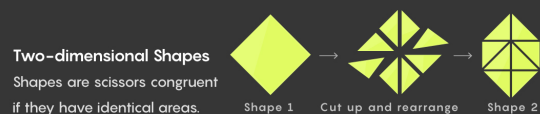
But for higher-dimensional shapes — like a three-dimensional ball, or an 11-dimensional doughnut that’s impossible to picture — the question of cutting one up and reassembling it as another is a lot harder. Despite centuries of effort, mathematicians have been unable to identify characteristics that dictate scissors congruence for most higher-dimensional shapes.

This fall, however, two mathematicians made the most significant headway on the problem in decades. In a paper presented at the University of Chicago on Oct. 6, Jonathan

*Campbell of Duke University and Inna Zakharevich of Cornell University took a substantial step toward proving scissors congruence for shapes of any dimension. ... In their effort to understand scissors congruence, Campbell and Zakharevich may have shown mathematicians a new way to think about a very different part of their field: algebraic equations. **FOR MORE, GO TO THE ARTICLE!***

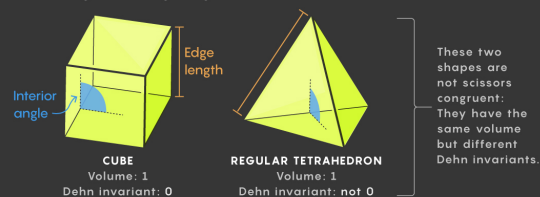
Scissors Congruence in Many Dimensions

Mathematicians want to know when shapes can be cut up and rearranged as each other. They hope the same properties that make shapes “scissors congruent” will help them understand algebraic equations.



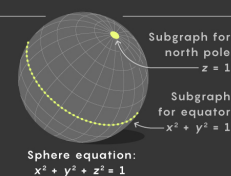
Three-dimensional Shapes

Higher-dimensional shapes are scissors congruent when they have the same volume and the same Dehn invariant — a value that reflects a shape’s interior angles and edge lengths.



From Shapes to Algebraic Equations

Just as shapes can be cut into pieces, graphs of algebraic equations can be broken into subgraphs. Mathematicians want to find an analogue of the Dehn invariant that indicates when two equations are made from the same pieces.



Reminders: Deadlines and Events

- **Hudson River Undergraduate Mathematics Conference (HRUMC).** This one-day math conference is **Saturday, April 4** at **Mount Holyoke College**.
 - If you are interested in presenting a talk, contact a math faculty member to sponsor it. The deadline for abstract submission is **Friday, March 6**, via the conference website: [HRUMC](#), linkable from the Union Math Department website -> Activities -> HRUMC -> Abstract Submission.
 - If you would simply like to attend this (free!) conference, contact **Professor Paul Friedman** (friedmap@union.edu) by **Friday, March 6**.
- **Steinmetz Symposium.** Planning on giving a talk at this annual rite of spring on Friday, May 8? Make sure to submit your abstract by the **Friday, March 6** deadline. The website for abstract submission is https://union-college.formstack.com/forms/steinmetz_submission

Problem of the Newsletter – March 2, 2020

Last week’s problem: Unfortunately, no correct solutions to last week’s problem were submitted. However, a solution has been posted at the newsletter sites in Bailey Hall.

This week’s problem: Here’s a little calculus problem to amuse you!

Find all differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f'(x) = \frac{f(x+n) - f(x)}{n}$$

for all real numbers x and all positive integers n .

Professor Friedman (friedmap@union.edu) will accept solutions until noon on Friday, March 6.