Department of Mathematics

April 6, 2020

Apart Together

During this time of COVID-19, while we are not on campus, we will still work to publish the math newsletter semi-regularly to provide you with information about happenings in the math department and the math community in general. In this way, we hope to maintain a sense of togetherness while we are apart.

Modeling COVID-19

As we follow the news of the spread of COVID-19, you might be wondering about the math behind the models used to predict what might be coming. As derivatives measure the rate of change of one variable with respect to another, it is no surprise that most models are based heavily upon differential equations. The "**SIR Model**," and modifications, is the classic one used to model the spread of disease. In this model, **S** represents the number of *susceptible* individuals, **I** is the number of *infected* individuals, and **R** stands for the number of *recovered* or *removed* individuals (either alive and immune, or dead). The system of differential equations in the model is as follows: with *a, b*, and *r* as constants,

$$\frac{dS}{dt} = -aSI$$
$$\frac{dI}{dt} = -aSI - bI$$
$$\frac{dR}{dt} = rI$$

The article on the second page of this newsletter is a primer on the SIR model, without much "math-speak." It is an excerpt from
https://www.wired.com/story/the-mathematics-of-predicting-the-course-of-the-coronavirus/.

The link about the SIR model in this excerpt takes you to the third page of a student-friendly teaching module "designed for use in first-semester differential calculus course to stimulate interest in the derivative as a tool for modeling rate of change," hosted by the Mathematical Association of America, and authored by David Smith and Lang Moore … from 2000 (before the Corona crisis). In this current era of online teaching, you are encouraged to work and think through this. A direct link is:
https://www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-introduction.

Finally, as much of our instruction this term is online via video, here are links to two different (but similar) video lessons about the SIR model. Both are quite good!

- Trefor Bazett's, "The MATH of Epidemics, Intro to the SIR Model,": https://www.youtube.com/watch?v=Qrp40ck3WpI. Trefor has a follow-up video, with variants of the SIR model
- Tom Rocks Maths, "Oxford Mathematician explains SIR disease model for COVID-19 (Coronavirus)," https://www.youtube.com/watch?v=NKMHhm2Zbkw. This takes the viewer, reasonably quickly, to a way to a formula for the maximum number of Infected persons.

About the SIR Model

The following is an excerpt from https://www.wired.com/story/the-mathematics-of-predicting-the-course-of-the-coronavirus/

***THE BASIC MATH** of a computational model is the kind of thing that seems obvious after someone explains it. Epidemiologists break up a population into "compartments," a sorting-hat approach to what kind of imaginary people they're studying. A basic version is an SIR model, with three teams: susceptible to infection, infected, and recovered or removed (which is to say, either alive and immune, or dead). Some models also drop in an E—SEIR—for people who are "exposed" but not yet infected. Then the modelers make decisions about the rules of the game, based on what they think about how the disease spreads. Those are variables like how many people one infected person infects before being taken off the board by recovery or death, how long it takes one infected person to infect another (also known as the interval generation time), which demographic groups recover or die, and at what rate. Assign a best-guess number to those and more, turn a few virtual cranks, and let it run.*

"At the beginning, everybody is susceptible and you have a small number of infected people. They infect the susceptible people, and you see an exponential rise in the infected," says Helen Jenkins, an infectious disease epidemiologist at the Boston University School of Public Health. So far, so terrible.

The assumption for how big any of those fractions of the population are, and how fast they move from one compartment to another, start to matter immediately. "If we discover that only 5 percent of a population have recovered and are immune, that means we've still got 95 percent of the population susceptible. And as we move forward, we have much bigger risk of flare-ups," Jenkins says. "If we discover that 50 percent of the population has been infected—that lots of them were asymptomatic and we didn't know about them—then we're in a better position."

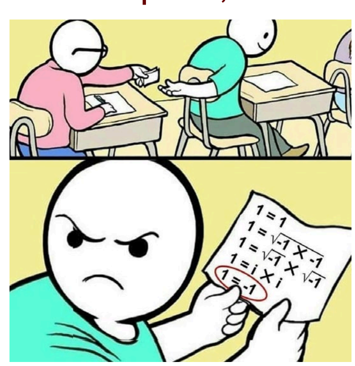
So the next question is: How well do people transmit the disease? That's called the "reproductive number," or R_0, and it depends on how easily the germ jumps from person to person—whether they're showing symptoms or not. It also matters how many people one of the infected comes into contact with, and how long they are actually contagious. (That's why social distancing helps; it cuts the contact rate.) You might also want the "serial interval," the amount of time it takes for an infected person to infect someone else, or the average time before a susceptible person becomes an infected one, or an infected person becomes a recovered one (or dies). That's "reporting delay."

And R_0 really only matters at the beginning of an outbreak, when the pathogen is new and most of the population is House Susceptible. As the population fractions change, epidemiologists switch to another number: the Effective Reproductive Number, or R_t, which is still the possible number of people infected, but can flex and change over time.

You can see how fiddling with the numbers could generate some very complicated math very quickly. (A good modeler will also conduct sensitivity analyses, making some numbers a lot bigger and a lot smaller to see how the final result changes.)

Problem of the Newsletter – April 6, 2020

This week's problem: Find the error in the "proof" that 1=-1.



Professor Friedman (friedmap@union.edu) will accept solutions until noon on Friday, April 10.