Department of Mathematics May 11, 2020

Petitioning Ends May 12

A reminder that the last day of petitioning for fall term classes is Tuesday, May 12, and that the following math courses are petition courses:

- All calculus courses numbered 115 and below, that is, Math 100, 110, 113, 115 and 115H
- Math 197 – Discrete Math for Computer Science
- Math 199 – Introduction to Logic and Set Theory
- Statistics 104 – Introduction to Statistics

Pieces from Theses and Senior Writing

This week, we have two contributions. The first is from **Bridget Logan**, who wrote her thesis under the direction of **Professor Paul Friedman**, and the second is from **Emily Rosenlof**, who participated in the Senior Writing Seminar with **Professor Brenda Johnson**.

As each academic year passed as a student at Union College, the daunting senior project was always at the back of my mind. As an underclassman, I always heard about the hours and weeks of work other students put into their project, and I was apprehensive about what a senior thesis had in store. Junior year, I decided that I would do a one-term math thesis for the winter term of my senior year. I had no idea what to expect, yet I knew it would not be an easy process. I was excited to be assigned to work alongside Professor Paul Friedman on this project, and my first assignment was pick from two topics. I was to do research about my chosen topic, and most importantly, go in a direction that got me excited to work on this project.

I eventually decided on Pascal's Triangle. I began my research during winter break, looking at countless articles about Pascal's Triangle and its applications. This process was challenging, as I had to read and pick through advanced, graduate level articles that I did not understand. However, one of the most important things my advisor taught me during the process of learning this material, was being able to read math. In other words, I read the articles and papers on this topic in depth, asking myself why and how something works, and even writing out some proofs in my own words in order to understand the material better. This is a skill I will be able to apply to courses I take in graduate school and in my future career.

While studying this triangle, I was also relearning background information and the building blocks of the triangle. Some of this information included binomial coefficients, the Binomial Theorem, Pascal's Formula, and more. I enjoyed this process because I was at liberty to study anything in relation to the triangle that interested me and got me excited to learn more about it. After more research and meetings, Professor Friedman and I decided to focus this thesis on properties of Pascal's Triangle and some generalizations.

At first, I was very interested in the patterns and properties within Pascal's Triangle. My research revolved around these patterns of the triangle, and eventually we looked at those same properties and patterns in a generalized Pascal's Triangle. One pattern I focused on was that by taking the sum of each ascending diagonal of Pascal's Triangle, we can find the Fibonacci numbers. Then, I applied this same method to a generalized Pascal's Triangle, known as an ab-based triangle in which a and b are nonnegative integers. In the ascending diagonals of the ab-based triangle, we have a Fibonacci-like recurrence sequence. By comparing Pascal's Triangle to the ab-based triangle, I was able to apply the theorems and definitions I already knew in general and abstract ways, thinking outside the box. I was also introduced to the combinatorial way of thinking throughout this process. A combinatorial interpretation is proven by counting the number of elements in a fixed set in two different ways to obtain the same answer. I learned how to prove the Binomial Theorem and Pascal's Formula combinatorically, and this allowed me to stray from the algebraic proofs I was so used to and think about these theorems in a different way.

What really helped in my research was being able to study this material on my own, and then bring back what

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I had learned to my advisor, so we could discuss it in our semi-weekly meetings. We discussed the material in great detail, and this allowed me to really understand, and then reiterate this knowledge in my own words in my thesis. During these meetings, my advisor had me try and teach him the information I had learned, as if he was trying to understand it for the first time. This was very difficult but explaining the information in my own words helped me to understand it even better.

During the term, time management was key. I had always struggled with time management but working on this thesis allowed me to improve that skill. By working on my thesis, a little bit every day, either learning the material, typing it up in LaTeX, or working through examples, I kept on track during the term. Only until the last couple weeks of the term was it time to get serious. At this time, I needed to organize my thesis, and explain each subtopic in specific mathematical language. Another very important skill I learned during this process was reading and typing the precise wording, mathematical terms and language that a math paper uses. I became much more comfortable writing this way, as well as writing and type setting in LaTeX.

This thesis project is by far the most challenging project I have done, yet it was a rewarding process and I learned more than I ever expected. I learned to manage my time, read and better understand advanced mathematical material, efficiently type on LaTeX, and successfully work with an advisor. With Professor Friedman's help and weeks of hard work, I successfully completed my thesis on time, and it is something I am extremely proud of. Constantly writing, typesetting, editing, reading, and then repeating that process all over again can be difficult at times, but I was able to push through because I was excited and interested in my research.

Emily Rosenlof wrote the following article about her senior writing experience.

To fulfill the senior writing component of my math major I decided to take the Senior Writing Seminar (MTH 487) with Professor Brenda Johnson. My decision to take this course was driven by my double major status which already included a formal thesis component in economics. Additionally, I had already taken Abstract Algebra with Professor Johnson and was interested in pursuing the topic further in the Seminar course.

Once the course had begun and we reviewed the basic concepts of group theory, I decided to individually study the topic of Puzzles. As a former Sudoku fanatic, I was intrigued when learning that I could use math to explain how different solved Sudoku puzzles were related. More than that, I found that I could use group theory to solve puzzles as complex as the Rubik's Cube. After reading multiple papers and textbook chapters that pertained to my topic, and consulting my four classmates and Professor Johnson, I narrowed the focus of my paper to include i) solvability criteria for two puzzles; the 15 Puzzle and the Oval Track Puzzle, and ii) a discussion of equivalence relations as they pertain to puzzle moves using mini-Sudokus and the Rubik's Slide puzzle.

Ten weeks and 15 pages later, I had completed the primary assignment of the course. Along the way I learned how to read high level math texts and journals, write my own words in a mathematical voice, and convey complex topics to an audience in a digestible way, all the while utilizing LaTeX, a typesetting system which was foreign to me before the seminar course. All of these accomplishments seemed unattainable, even comical, when beginning the journey ten weeks prior. Having the constant support of my "seminar"-mates, whose roadblocks often paralleled my own, and consistently meeting one-on-one with Professor Johnson were critical components of my success.

I would wholeheartedly recommend the seminar course to an underclassmen math major. There is great learning and growth potential in such a small group setting, something that is harder to come by in an individual thesis. That being said, the final decision should first and foremost reflect your mathematical interests. This is a project that is just as challenging as it is time consuming, so you want to ensure that you're invested in and able to enjoy the journey.