Department of Mathematics October 12, 2020

Mathematician/Mathematical Physicist Awarded Nobel Prize in Physics

On October 6, 2020, the Royal Swedish Academy of Sciences awarded one half of the 2020 Nobel Prize in Physics to **Sir Roger Penrose** of the University of Oxford *"for the discovery that black hole formation is a robust prediction of the general theory of relativity"*, with the other half awarded jointly to **Reinhard Genzel** of the Max Plank Institute in Germany and the University of California, Berkeley, and **Andrea Ghez** of the University of California, Los Angeles, *"for the discovery of a supermassive compact object at the center of our galaxy."*

Sir Roger Penrose
(photo from Wikipedia)

Roger Penrose is a renowned mathematician: per his Wikipedia page, his 1958 PhD thesis was "on 'tensor methods in algebraic geometry' [written] under algebraist John A. Todd." He has contributed groundbreaking work in pure mathematics, including the creation of so-called *twistor* theory. But his scientific interests are wide, and they led him to mathematical physics and the study of general relativity and black holes.

Union College's Math Department has some loose connections to Penrose. **Professor Christina Tønnesen-Friedman** is Penrose's academic "grandchild": Penrose supervised Claude LeBrun, who supervised Professor Tønnesen-Friedman's PhD work. In addition, **Professor Jeff Jauregui**'s research has been influenced by Penrose. Professor Jauregui shares his reflections on Penrose's work below. (Thanks, Jeff!)

On Penrose's Nobel Prize Work, by Jeff Jauregui

I was very excited to learn of Sir Roger Penrose being a recipient of this year's Nobel Prize in physics! His accomplishments and contributions to physics and mathematics have been vast. Below I try to describe a bit about the part of his work that was primarily recognized by the Nobel Prize committee, and also share how some of my own research has been influenced by his work.

Einstein's theory of general relativity (GR) was a revolutionary approach to describing gravity and the physics of the universe on the large scale. The complete theory was presented by Einstein in the fall of 1915, and famously implies that the force of gravity manifests as the curvature of the 4-dimensional spacetime in which we live. The first nontrivial solution of Einstein's equation was found by Karl Schwarzschild the same year (while he was serving in the German army in World War 1). It modeled the gravitational field of a rotationally symmetric, static object, such as a star. However, the solution had a curious feature that when the radial coordinate was small, the solution would "blow up." For decades this anomaly was dismissed as a mathematical artifact with no physical meaning. Over time, the concept of a "black hole" emerged, with Schwarzschild's solution being the simplest example. However, it remained unclear whether black holes were mathematically permitted to form dynamically in Einstein's theory. Furthermore, even if the math allowed it, *did* they actually exist in our universe? And was there truly a "singularity" at the center of a black hole, where space and time come to an end?

In the 1960s Penrose proved a "singularity theorem," showing that from the point of view of Einstein's equation, singularities can and do form, and in many cases are inevitable. The statement says that under reasonable physical conditions (along the lines of nonnegative energy density), if a spacetime contains a "trapped surface", then there must exist a path in the spacetime that a particle could take that is "incomplete" - meaning it reaches an end in the spacetime after traveling some finite length. Trapped surfaces themselves form when enough mass is condensed into a small enough space (such as when a large star exhausts its fuel and collapses under its own gravity). These theorems presented a very strong theoretical case for the existence of black holes (if you assume Einstein's theory is correct). Conversely, if black holes could be detected, that would present strong evidence for the validity of GR!

Much, much more is known about black holes today, including the fact that they exist all throughout our universe. In fact, the other half of the 2020 Nobel Prize in physics went to a team (Reinhard Genzel and Andrea Ghez) who used years of observational data to show that a supermassive black hole lives at the center of our own galaxy! (How massive? As much mass as *millions* of our own sun!). **(CONTINUED on P2.)**

Penrose has many other contributions just within the field of GR. One of these is the formulation of the "Penrose conjecture." This predicts a deceptively simple statement, that the total mass of a spacetime is at least the mass of all the black holes contained within it. In its full generality, this has remained a difficult unsolved problem for almost 50 years! My own Ph.D. thesis advisor, Hugh Bray of Duke University, proved the Penrose conjecture in the case of "time symmetry" in 2001, which is a major result within the field of differential geometry. This played an important role in my dissertation and some of my subsequent papers. Incidentally, I had the opportunity to see Penrose speak in 2007!

If you'd like to hear more, please feel free to reach out to me. Some suggested reading:

- Black holes and time warps, Kip Thorne (also a Nobel laureate)
- Einstein's War, Matthew Stanley (great account of how GR was developed amidst World War I)

Winter Term Preregistration: Petitioning Begins this Weekend

The Petition Math Courses: IMP 120; Math 110, 113, 340, and 487; Stats 104

The petitioning process for winter term classes begins Saturday, October 17, and runs through Tuesday, October 20. If you are interested in taking any petition courses, make sure to submit a petition through WebAdvisor. After faculty have reviewed the petitions, students then need to accept or decline the petitions that they were offered; this is to be done between Tuesday, October 27 and Thursday, October 29.

The Courses: This winter, the Math Department is offering several interesting courses beyond the calculus sequence that are suitable for math majors and minors.

Math/Stats 128 is a calculus-based introduction to probability. Students who might be interested in a career as an actuary or in financial mathematics should consider this course. This course is also helpful for economics majors, statistics minors, and prospective teachers.

Math 199 is the department's "bridge course," intended to help students make the transition from computationally oriented courses to more theoretical proof-writing courses. It is a **required** course for all math majors and minors that is *usually* taken *after* a student has taken Math 115.

Beyond Math 199: There are several courses being offered in the winter that have a Math 199 prerequisite:

- **Math 234** (Differential Equations). This course takes a somewhat more theoretical approach to the study of differential equations than its 100-level counterpart, Math 130. Note that students may only take one of these two courses.
- **Math 235** (Number Theory). In this course, properties of the integers are studied, from divisibility and modular arithmetic through Gauss' crowning achievement of Quadratic Reciprocity. MTH-235 normally is closed to students who have passed MTH-221 or MTH-054.
- **Math 340** (Linear Algebra). This is a foundational course in math that is **required** for math majors. The primary objects of study in this course are vector spaces and the linear maps between them.

Statistics 264 (Regression Analysis). In this course, both the theory and application of regression analysis to develop regression models to fit real-world data sets are studied. Prerequisite(s): MTH 115 and one of STA 104, ECO 243, STA 164, PSY 200, ECO 243, MER 301 or permission from Chair.

Putnam Exam Preparation – fun math problem solving sessions
Thursdays, 7:00-8:00pm in Bailey 207.
Contact: Prof. Greg Malen (maleng@union.edu)

Math Club Movie Night: Thursday, October 15.
Contact Lily Dong (dongl@union.edu) for more information.