

## UNDERGRADUATE MATH SEMINAR

The math seminar will be emerging from its COVID induced hibernation! There are three seminars scheduled for the spring term, one during each of Weeks 3, 6, and 9. They are presently planned to be on Thursdays during the common lunch hour, 1:20 – 2:15, and will be held via Zoom.

The first seminar of the term will be

**DATE:** **Thursday, April 15**

**Time:** **1:20 – 2:15**

**Zoom:** <https://union.zoom.us/j/92591304428>

In this week's seminar, **Emily Hoopes-Boyd** from **Kent State University** will be delivering the following talk:



Emily Hoopes-Boyd  
Kent State University

### Title: The Images of Polynomials Evaluated over Matrices

**Abstract:** A version of the L'vov-Kaplansky conjecture states that the image of a multilinear polynomial evaluated over matrices, with entries from the complex numbers, is a vector space. This statement has been proven only for the case of  $2 \times 2$  matrices, but many partial results have been proven within the last decade. We will consider this problem in a slightly different context; rather than taking the matrix entries to be from the complex numbers, we will consider matrices over an algebraically closed skew field, which we will denote by  $K$ . We will show that the image of any multilinear polynomial with coefficients from  $K$ , evaluated over  $M_m(K)$ , is  $M_m(K)$ . We will also prove that any matrix in  $M_m(K)$  may be written as the sum of three or fewer elements from the image of any generalized polynomial. In particular, the image of the polynomial  $xy-yx$  has some special properties over a variety of matrix rings, including the ring of matrices over the complex numbers.

## Senior Writing and Pieces from Thesis

**Celine Nguyen** wrote her two-term senior thesis this past fall and winter terms under the direction of Professor Christina Tønnesen-Friedman.

I finished my two-term thesis just a few weeks ago under the supervision of Professor Christina Tønnesen-Friedman. If I were able to change anything about it, I would prefer to be on-campus and talk with my advisor in her office in Bailey Hall, instead of meeting biweekly via Zoom. Other than that, I have no regret about all the decisions I made along my thesis journey and I would not do anything differently. I would like to thank Professor Tønnesen-Friedman for making my writing thesis journey an unforgettable, exhilarating, and rewarding experience. It is undoubtedly a perfect way to bring my undergraduate studies to an end.

My thesis focuses on the Problem of Thirteen Spheres, which asks whether 13 equal-sized non-overlapping spheres in three dimensions can touch another sphere of the same size. At first, the problem seemed evident and quite effortless, but its proof requires a solid foundation from spherical geometry. Unlike Euclidean (plane) geometry which deals with points and lines on flat surfaces, spherical geometry plots points and great circles on the surface of a sphere. It was a challenging but enjoyable process of visualization and developing new concepts, for example, in spherical geometry three angles of a triangle do not add up to  $180^\circ$ .

I also studied a simpler version of sphere packing, which is its two-dimensional version, circle packing. This geometry problem is incredibly relevant to social distancing during the COVID-19 pandemic. In order to keep at least two meters away from each other, each person needs to be at the center of a circle of radius one meter, where the circles cannot overlap. Thus, solving the questions regarding the maximum number of people to be allowed in a certain area amounts to fitting as many non-overlapping equal-sized circles into a confined

floor plan as possible. I calculated the density of square packing as well as hexagon packing of circles and went over Thue's theorem, which says that hexagon packing is the optimal packing of circles.

Writing a thesis was a rewarding opportunity for me to explore a deep and fascinating topic, and I would recommend this experience to every underclassman. You may have known that choosing a topic is important and that you should pick one that is manageable and of your interest. I would suggest you take various math courses to see what area you like best. Also, the sooner you reach out to your advisor, the better. I actually switched my topic during the summer after junior year. Finally, manage your time wisely and plan how you want your thesis to be in advance. I hope you enjoy the process of writing a thesis as much as I did!

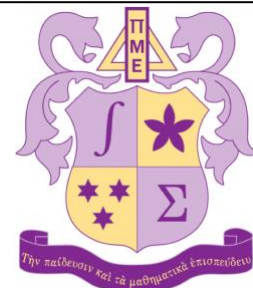
## Pi Mu Epsilon – Math Honors Society Accepting Applications

Pi Mu Epsilon (PME) is a national undergraduate math honors society. In Spring 2013, a chapter of PME was established at Union College. Now we are looking to induct new members who have demonstrated an ability and interest in mathematics consistent with PME's purpose and goals.

To be considered for membership into Union's chapter of PME, a student must have

- taken at least two math courses at the 200-level or above;
- a minimum campus-wide GPA of 3.0 and a minimum GPA of 3.0 in all math courses that can be used to fulfill a math major;
- a demonstrated engagement and involvement, while at Union College, in the promotion of mathematics at Union College and/or broader community, (examples include, but are not limited to, active involvement in the Union College Math Club, in the Association of Women in Mathematics, in the Hudson River Undergraduate Mathematics Conference, in the Calculus Help Center, in math tutoring through the Kenney Center; regular attendance at the Union College Math Department Seminar);
- an interest in joining PME, demonstrated by writing a few paragraphs/mini-essay in an email to **Professor Paul Friedman** ([friedmap@union.edu](mailto:friedmap@union.edu)) explaining your interest, your qualifications, and how you have supported and will continue to support the purpose and goals of PME. This email must be received by **noon on Friday, April 23<sup>rd</sup>**.

The national initiation fee for PME is \$30. This covers the cost of membership and also a one-year (two issue) subscription to the MPE Journal. Members of MPE can also purchase other PME items such as tassels for graduation, t-shirts, etc. For more information, go to the website [pme-math.org](http://pme-math.org).



## HRUMC: Kudos to the Union Speakers

On Saturday, April 10, three people from Union, **Professor Louisa Catalano**, **Dante Scott '21**, and **Jason Stack '21**, gave talks at the Hudson River Undergraduate Mathematics Conference. Congratulations!

The first screenshot shows a presentation titled "Many Proofs of the Infinitude of Primes" by Danté Scott, presented at HRUMC on April 10, 2021. The second screenshot shows a presentation titled "All Jordan Type Partitions for a Hilbert Function of the Complete Intersection Type" by Jason Stack, presented at Union College in April, 2021. The third screenshot shows a presentation titled "On maps preserving products" by Louisa Catalano, presented at Union College on April 10, 2021, as a joint work with Megan Chang-Lee (Brown University) and Hayden Julius (Kent State University).