Department of Mathematics
November 1, 2021

UNDERGRADUATE MATH SEMINAR

Professor Joe Chen

The next math seminar will be at a **different day and time** than usual!

DATE: **FRIDAY, November 5**

Time & **Location:** **4:00pm –** Refreshments in **Bailey 204**

4:30 – 5:20pm: Seminar in **Bailey 207**

In this seminar, **Professor Joe Chen** from the Department of Mathematics at **Colgate University** will deliver the following talk:

Title: Solving Probability Using Differential (In)equations!

Abstract: In calculus or analysis we learned the notion of convergence, $\lim_{n\to\infty} x_n = x$. It would be even better if we can quantify the rate of convergence, namely, how fast does $|x_n - x|$ decay to 0 as $n \to \infty$? Exponential? Or as a power law? Now translate these questions in the setting of your favorite probabilistic model (random walks, Langevin diffusion, etc. Or you can use last week's talk as an example). All you know is that the model converges to a stationary distribution as time $n \to \infty$. But how fast does it converge? And since the model might be idealized, can you come up with a real-world algorithm that approximately samples this stationary distribution? And how good is the approximation? One answer to this question can be obtained through the analysis of a first-order-in-time differential inequality called the "entropy inequality." After sketching the basic properties of relative entropy, I will explain the structure of the entropy inequality, and illustrate its use in recent research in math and computer science.

A Tidbit on Stirling's Formula

Stirling's Formula is one of several cool and remarkable formulas in mathematics that relate several important and interesting numbers in unexpected ways. One version of Stirling's formula is

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

This formula tells us that factorials can be approximated using pi and e (as well as square roots and powers). Crazy! Is there any real-world meaning to this? In a recent blog post, John Carlos Baez, a mathematician at the University of California, Riverside, provides an explanation below. For more information, read the full post at https://johncarlosbaez.wordpress.com/2021/10/08/stirlings-formula-in-words/)

STIRLING'S FORMULA:
WHAT DOES IT REALLY MEAN?

Suppose raindrops are randomly landing on your patio at a constant average rate that you know. You start your stopwatch, wait until the *expected* number of drops that have landed is n, and record how many have *actually* landed. Do this many times and get a probability distribution of answers. As $n \to \infty$, this probability distribution approaches a Gaussian with mean n and standard deviation $\sqrt{n}$.

Translate this fact into equations and you get Stirling's formula:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

n	Factorial	Stirling
1	1	0.9
2	2	1.9
3	6	5.8
4	24	23.5
5	120	118.0
6	720	710.1
7	5040	4980.4
8	40320	39902.4
9	362880	359536.9
10	3628800	3598695.6