Department of Mathematics

February 21, 2022

Undergraduate Math Seminar

This seminar is on hiatus this week. We look forward to seeing you in the coming weeks.

Pieces from Thesis

Adam Ginsberg wrote his senior thesis this past fall term under the direction of Professor Louisa Catalano.

My thesis, Using Mathematical Functions to Express Harmonic Progression, explored a system designed to connect chords and harmonic progression in music to arithmetic functions in mathematics. I became inspired to work on this topic when I first discovered an article by Sidarth Jayadev baselining a way to map any one chord to another using matrices, creating a matrix equation that expressed the relationship between the two chords. This process was promising but had its flaws. The matrix equation was long, tedious, and sometimes unnecessary to calculate. I sought to find a less complex process, including a simpler set of functions to use.

I focused my studies on the twelve-tone chromatic scale found commonly in European Classical music through modern pop music. By mapping each unique chord tone, or note, to an integer, we can create three- or four-note integer sets that represent a musical chord. The tonic, or initial note of the scale, has a value of zero and every raised semitone increases the integer value by one. Each octave is displaced by twelve semitones, with each note of an octave being the same value. Thus, much of my thesis used only the integers zero through eleven, working modulo 12.

With the baseline system explained, I then explored patterns in simple arithmetic equations that mapped chords to one another. In the example of f(x) = x+c, where *c* is a constant, we show that any chords of the same chord quality can map to each other. In another example, f(x) = 11x+c often mapped major chords to minor chords, and vice versa. I sampled from over 500 functions, including the Sum of Squares Function, Euler's Totient Function, Ramanujan's Tau Function, and various additive, subtractive, multiplicative, exponential, and trigonometric functions. Yet this system was not perfect, as there were sometimes two note sets that could not easily map to one another. In that case, the aforementioned matrix equation became the last resort to create a function. Using these functions, we can create 'function sets' which portray the harmonic progression of full songs.

I really enjoyed being able to explore a topic of my choice, and I am extremely thankful to Professor Catalano for allowing me to have the academic freedom to research this topic. I am double majoring in Mathematics and Music, and in the field of music I really enjoy theory and composition. It was a unique experience to be able to take my background knowledge in music theory and apply it to mathematics, while learning new material in the fields of modular arithmetic, number theory, and matrix theory.

For incoming seniors, my greatest piece of advice is to choose a topic that you are passionate about or is relevant to you. The process will be much more rewarding if you pick a topic you enjoy. Talk with professors if you have your own idea for a thesis, as the field you may be looking to research might line up with one of their fields of study. In addition, keep open communication with your advisor. Whether you are overjoyed, struggling, staying ahead or falling behind, let your advisor help you. Your thesis should be one of the greatest research papers you write in college. If done right, a thesis can truly be special.

The next meeting of the Math Club will **Thursday, February 24 at 6pm Math Common Room – Bailey 204** Elections for leadership positions will be held.