Department of Mathematics

UNDERGRADUATE MATH SEMINAR

The next **two** math seminars are announced below. One is at the latter part of this week, and the other is at the beginning of next week. Details of the seminars will be announced on posters in Bailey Hall. We hope to see you there!

- Thursday, March 7, 4:15-5:15pm in Bailey 207
- Monday, March 11, 4:15-5:15pm in Bailey 207

Hungry for Math? How about an "Entropy Bagel"?

The latest edition of the online math and science magazine, <u>Quanta</u> <u>Magazine</u>, has a fantastic article, <u>"Entropy Bagels' and Other</u> <u>Complex Structures Emerge from Simple Rules</u>" by Jordana Cepelewicz leading readers from an overview of a basic *dynamical system* to some beautiful fractal rings that arise in the complex plane dubbed entropy bagels.

Starting from a simple quadratic, $f(x) = x^2 + c$ where *c* is a complex number, and iterating its values starting with x = 0, the critical point of *f*, can lead to some wonderfully interesting questions and still not completely understood phenomena. For example, does the sequence that arises become a repeating, or almost repeating, cycle, or does it produce "chaotic" behavior? "For [real] values of *c* smaller than -2 or bigger than 1/4, iteration quickly blows up to infinity. But within that interval, there are infinitely many values of *c* known to produce chaotic behavior, and infinitely many cases like -3/2, where "we don't know what happens, even though it's super concrete," said Giulio Tiozzo of the University of Toronto."

Further, "[t]here are infinitely many real quadratic equations that, when iterated from zero, are known to end up producing a repeating cycle of numbers. But if you restrict *c* to rational values — those that can be written as fractions — only three values eventually generate periodic sequences: 0, -1 and -2."

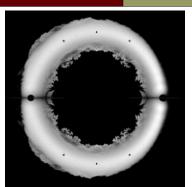
More recent explorations introduce *entropy*, a measure of the difficulty of predicting the sequence of numbers resulting from iterations of $x^2 + c$. Combining this with another ingredient, Galois conjugate roots of a polynomial a recipe, leads one to entropy bagels! Intrigued? Then read the <u>full article</u> in Quanta!

Reminders

- Sign-up for the Hudson River Undergraduate Mathematics Conference (HRUMC), being held on Saturday, April 6 at Keene State College: contact Professor Paul Friedman (<u>friedmap@union.edu</u>) by March 15.
- The Calculus Help Center (CHC) is still open: Sunday through Thursday nights, 7:30-10:00pm in the Sorum House seminar room. The CHC's last night is on the last day of classes: March 12.

Problem of the Newsletter: Kudos!

Congratulations to Union College librarian John Myers for correctly solving last week's problem! Nice work!



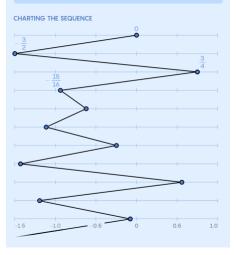
An entropy bagel!

Stability or Chaos?

Mathematicians still don't know what happens when you iterate the simple equation $f(x) = x^2 - 3/2$. The sequence you produce might eventually end up almost repeating, or it might endlessly bounce around without ever settling into an orderly pattern.

| REPEAT, REPEAT | To iterate: $f(x) = x^2 - \frac{3}{2}$ |
|------------------------------|---|
| 1 Start with $x = 0$. | $\longrightarrow \left(0\right)^2 - \frac{3}{2} = -\frac{3}{2}$ |
| 2 Set x equal to the output. | $\left(-\frac{3}{2}\right)^2 - \frac{3}{2} = \frac{3}{4}$ |
| 3 And repeat. | $\left(\frac{3}{4}\right)^2 - \frac{3}{2} = -\frac{15}{16}$ |

4 Nobody knows what happens when you keep repeating this process indefinitely.



March 4, 2024