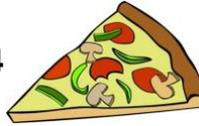## UNDERGRADUATE MATH SEMINAR

The next seminar of the winter term will be

DATE: <mark>**Thursday, February 19**</mark>

Time &   **12:30 –** Refreshments in **Bailey 204**

Location: **12:50 – 1:45** Seminar in **Bailey 207**

Seminar details will be posted around Bailey Hall and on the [Math Department's website](#) soon.

Pieces from Thesis: Ananya Gupta

*Ananya wrote her senior thesis this past fall under the guidance of **Professor Phanuel Mariano.***

Optimizing Allocation of Public Resources Under Uncertainty: A Kelly Type Approach. How should governments allocate public resources when outcomes are uncertain? Traditional economic models often assume that public investments in education, infrastructure, or healthcare produce predictable results. In reality, however, fiscal outcomes depend on changing political conditions, implementation quality, and institutional efficiency. The same investment may produce very different results across time.

In my senior thesis, I explore this question through a mathematical lens by combining ideas from probability, dynamic systems, and matrix theory. My thesis investigates how uncertainty affects long-run economic growth and how mathematics can help us describe optimal decision-making in such environments.

A Kelly-Type Perspective. The mathematical inspiration for the project comes from the Kelly criterion, a famous strategy originally developed to maximize long-term growth in betting and investment problems. Rather than focusing on short-term gains, the Kelly approach asks: *what strategy maximizes growth over time when outcomes are random?*

Public investments can be viewed similarly. Governments allocate resources today without knowing whether spending will be highly effective or poorly implemented. Over time, these outcomes compound multiplicatively, much like returns on an investment portfolio. To model this process, the economy is described as evolving through a stochastic system that alternates between high-efficiency and low-efficiency states. These fluctuations are modeled by a binary random variable that takes two efficiency outcomes with probability $p=1/2$, and this variable determines the random component of the transition matrices. The resulting dynamics are captured through products of random matrices.

The Role of the Lyapunov Exponent. The central mathematical object in the project is the *Lyapunov exponent*, which measures the long-run exponential growth rate of a system, over time (n). In this context, it represents the economy's expected long-term growth under repeated efficiency shocks. Computing Lyapunov exponents exactly is notoriously difficult. To overcome this, we use a recursive method that constructs explicit upper and lower bounds for the exponent. These bounds converge toward the true value, allowing us to approximate long-run growth behavior without requiring a closed-form solution.

The mathematics involves affine recursions, invariant distributions, and carefully structured inequalities that control how uncertainty propagates through the system. My main theorems for the Lyapunov exponent and its existence are given by:

Theorem 1. *The Lyapunov exponent $\gamma(p)$ can be estimated by $\gamma(p)_n^- \leq \gamma(p) \leq \gamma(p)_n^+$, where*

$$\gamma(p)_n^- := \frac{1}{n}\log\left(\frac{1}{\delta}\right) + \frac{1}{n2^n}log(c_n)$$
$$\gamma(p)_n^+ := \frac{1}{2n+2}\log\left(\frac{1}{\delta}\right) + \frac{1}{(n+2)2^n}log(c_n).$$

Lemma 2. *Suppose $\gamma(p)_n^- \leq \gamma(p) \leq \gamma(p)_n^+$ holds, then $limsup|\gamma(p)_n^+ - \gamma(p)_n^-| = 0$.*

Mathematics Beyond Theory. Although the paper itself is largely theoretical, what truly brought the project to life for me was thinking about applications. Drawing on my background in economics, I began imagining scenarios where such an optimization problem could arise in real policy decisions. Public spending under uncertainty is one such example, where policymakers must balance risk, efficiency, and long-term growth outcomes, even though many simplifying assumptions are required to make the mathematics tractable.

For anyone about to begin the thesis process, I know it can feel daunting at first. I certainly would not have imagined myself writing a pure math thesis when I first entered college, but I realized along the way that the courses we take really do prepare us more than we think. If I could go back to the beginning of the process, I would tell myself to trust it and to remember that your advisor genuinely wants to help you succeed. I began working on my thesis as part of summer research before it officially became my thesis project, and at the time, much of the mathematics felt unfamiliar and intimidating. Slowly, though, the ideas started to come together.

Ultimately, this project reminded me how powerful mathematical modeling can be: abstract tools such as random matrices and Lyapunov exponents may seem distant from everyday problems, yet they provide a language for thinking rigorously about uncertainty.

Mark Your Calendar: HRUMC 2026, April 25 at Vassar College

The 32nd annual Hudson River Undergraduate Mathematics Conference (HRUMC) will be **Saturday, April 25** at **Vassar College** in Poughkeepsie, NY. This is a *free* daylong conference, and the math department will provide transportation for those interested in attending.

The conference features short, 15-minute talks *primarily by students* along with some by faculty, as well as a keynote address by a prominent mathematician. This year's keynote will be *"Topological Time Series Analysis,"* by **Jose Perea** of **Northeastern University.**

- If you are interested in **presenting a talk** (based on summer research, a thesis, a project, or problem you enjoyed, etc.) contact a math faculty member to sponsor it.
- If you are interested in **attending the conference or knowing more about it**, email **Professor Paul Friedman** (friedmap@union.edu).

Conference registration is now open on the HRUMC website, HRUMC (also findable from Union Math Department website -> Activities -> HRUMC). Registration deadlines are in early April.

Information about the conference will continue to be advertised in this newsletter and around Bailey Hall.